## This packet is intended for students going into

## Course 3

Directions: Complete the following math packet week by week. Each week you will find the topic divided into parts so you can manage the workload. This packet has 6 weeks of materials. Take your time and avoid the summer slide by completing the following work that will prepare you for Course 3. Additionally, at the end of each section, you will find a "Minute math" activity. These problems are designated to improve your math fluency and practice using strategies for solving a variety of problems.

## Week 1:

- Adding, Subtracting, and Multiplying Fractions - Reciprocals
- To add fractions that have the same denominators, add the numerators. The denominator does not change.

Example: $\frac{3}{5}+\frac{2}{5}=\frac{5}{5}=1$

- To subtract fractions that have the same denominators, subtract the numerators. The denominator does not change.

Example: $\frac{5}{9}-\frac{1}{9}=\frac{4}{9}$

- To multiply fractions, multiply across both numerators and denominators.

Example: $\frac{1}{2} \rightrightarrows \frac{3}{4}=\frac{3}{8}$

- To find the reciprocal, "flip" (reverse the terms of) the fraction.


The product of a fraction and its reciprocal is 1.

Example: $\frac{3}{4} \cdot \frac{4}{3}=\frac{12}{12}=1$

- The Inverse Property of Multiplication says that a $\frac{1}{a}=1$ if $a$ is not 0 .


## Practice:

Simplify 1-3.

1. $\frac{7}{8}+\frac{1}{8}$
2. $\frac{3}{4} \cdot \frac{1}{3} \cdot \frac{2}{5}$
3. $\frac{5}{7}-\frac{1}{7}$

Write the reciprocal of each fraction.
4. $\frac{3}{10}$
5. 6
6. $\frac{9}{5}$

Find the value of each unknown number.
7. $\frac{9}{12} a=1$
8. $\frac{6}{5} b=1$
9. $12 c=1$

- Equivalent Fractions
- Reducing Fractions, Part 1
- Equivalent fractions have the same value.
(Equivalent means "equal.")
Form equivalent fractions by multiplying any fraction by another fraction equal to 1 .
Example: $\frac{3}{4} \times \frac{3}{3}=\frac{9}{12}$

$$
\frac{9}{12} \text { is equivalent to } \frac{3}{4} \text {. }
$$

- Reduced fractions are also called equivalent fractions.

Reduce fractions by dividing any fraction by another fraction equal to 1 .
Example: $\frac{9}{12} \div \frac{3}{3}=\frac{3}{4}$

$$
\frac{9}{12} \text { reduces to } \frac{3}{4} \text {. }
$$

The terms of a fraction are the numbers used when writing a fraction. Reduce fractions to lowest terms by dividing the terms by the GCF. If both terms of a fraction cannot be divided by the same number, the fraction cannot be reduced. For example, the fractions $\frac{2}{5}$ and $\frac{4}{9}$ cannot be reduced because they are already in lowest terms.
Reading a ruler requires knowing how to reduce fractions.


## Practice:

Reduce each fraction.

1. $\frac{8}{24}$
2. $\frac{5}{25}$
3. $6 \frac{2}{6}$

Complete each equivalent fraction.
4. $\frac{4}{9}=\frac{?}{54}$
5. $\frac{3}{7}=\frac{15}{?}$
6. $\frac{10}{45}=\frac{?}{9}$
7. Which of the following does not equal $2 \frac{1}{5}$ ?
A. $\frac{33}{15}$
B. $\frac{12}{5}$
C. $\frac{11}{5}$
D. $2 \frac{2}{10}$

Part 3:

- Exponents
- Rectangular Area, Part 1
- Square Root
- An exponent shows repeated multiplication.

It shows how many times the base is used as a factor.
Example: base $\longrightarrow 5^{4} \longleftarrow$ exponent

- To calculate with exponents, write the products.

Examples: $5^{2}+3^{2}=5 \cdot 5+3 \cdot 3=34$

$$
2^{3} \cdot 2^{2}=2 \cdot 2 \cdot 2 \cdot 2 \cdot 2=32
$$

$$
7^{3} \div 7^{2}=\frac{7 \cdot 7 \cdot 7}{7 \cdot 7}=7
$$

- "Cover" is the keyword for area.

$$
\text { Area }=\text { length } \times \text { width }
$$

Label area with square units.
Example:


- To find the square root of a number, find the factor that was multiplied by itself.

Square root symbol:
This symbol is read as "the square root of." $\sqrt{25}=5$ is read as "the square root of twenty-five equals five."
Squaring and finding the square root are inverse operations. One operation "undoes" the other operation.


Each side is 5 . $5^{2}=25$ squares

The square root of 36 is 6 .


Each side is 6.
$6^{2}=36$ squares

$$
8^{2}=64 \text { and } \sqrt{64}=8 \quad\left(\frac{1}{4}\right)^{2}=\frac{1}{4} \cdot \frac{1}{4}=\frac{1}{16} \text { and } \sqrt{\frac{1}{16}}=\frac{1}{4}
$$

## Practice:

1. A rectangle is 30 ft long and 10 ft wide. What is the area of the rectangle?

Simplify 2 and 3.
2. $10^{2}-2^{3}-\sqrt{49}$ $\qquad$
3. $\left(\frac{9}{10}\right)^{2}$
4. What two facts about squares and square roots does this figure illustrate?


## Minute Math Week 1:



1. Simplify: $12(2+7+1)=$
2. $\frac{3}{10} \cdot \frac{7}{10}=$
3. Circle all of the following equal to $\frac{2}{5}: 0.4 \quad \frac{4}{100} \quad 40 \%$

4. $10 \cdot \square=5$
5. Cross out the three-dimensional shape. $\square$

6. Each side of the regular pentagon is 5 centimeters. What is the perimeter? $\qquad$

7. In the graph, Alex has $\qquad$ times as much money as Annie.
8. If $a=5$ and $b=4$, then $2 a+b=$ $\qquad$ .
9. If $3 x=27$, then $x=$ $\qquad$

10. Which of the following shapes comes next in the pattern?

a. $\square$
b.

c.

d.


## Week 2:

Part 1:

## - Dividing Fractions

- The reciprocal of any whole number or fraction is the reverse of terms. The product of a term and its reciprocal is 1 .

Examples: The reciprocal of 3 is $\frac{1}{3}$.
The reciprocal of $\frac{1}{3}$ is 3 , or $\frac{3}{1}$.
The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.
The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

- To divide whole numbers sometimes multiply by the reciprocal of the divisor.

Example: $24 \div 3=\frac{24}{1} \div \frac{3}{1}=\frac{24}{1} \times \frac{1}{8}=8$

- To divide fractions, multiply by the reciprocal of the divisor.

Example: $\frac{2}{3} \div \frac{1}{3}=\frac{2}{\nmid} \times \frac{1}{1}=2$

1. Copy the first number (fraction).
2. Change $\div$ to $\times$.
3. Write the reciprocal of the second fraction.
4. Cancel (reduce pairs) if possible.
5. Multiply.


## Practice:

Simplify 1-6.

1. $\frac{3}{16} \div \frac{1}{8}$
2. $\frac{2}{3} \div \frac{3}{4}$
3. $\frac{8}{15} \div \frac{1}{3} \longrightarrow$
4. $\frac{1}{5} \div \frac{7}{10}$
5. $12 \div \frac{1}{6}$
6. $\frac{6}{7} \div \frac{12}{14}$

Part 2:

## - Multiplying and Dividing Mixed Numbers

- To multiply and divide mixed numbers:

1. Change mixed numbers to improper ("top heavy") fractions.
2. Then multiply or divide.
3. Simplify (reduce and/or convert) as necessary.

## Examples:

| Multiply |  | Divide |
| :---: | :---: | :---: |
| $2 \frac{1}{2} \times 1 \frac{2}{3}$ | $3 \frac{1}{3} \div 2 \frac{1}{2}$ |  |
| $\downarrow \downarrow$ | $\downarrow \downarrow$ |  |
| $\begin{array}{ll} \frac{5}{2} \times \frac{5}{3} & \begin{array}{l} \text { Change mixed } \\ \text { numbers to improper } \\ \text { fractions. } \end{array} \end{array}$ | $\begin{gathered} \frac{10}{3} \div \frac{5}{2} \\ \end{gathered}$ | Change mixed numbers to improper fractions. |
| $\frac{5}{2} \times \frac{5}{3}=\frac{25}{6}$ Multiply. | $\frac{10}{3} \times \frac{2}{5}=\frac{4}{3}$ | Multiply by reciprocal of the divisor. |
| $=4 \frac{1}{6}$ Simplify. | $=1 \frac{1}{3}$ | Simplify. |

## Practice:

Simplify 1-6:

1. $4 \frac{1}{2} \times \frac{1}{3}$
2. $1 \frac{1}{4} \cdot 7 \frac{3}{5} \cdot 2 \frac{1}{2}$
3. $6 \times 4 \frac{1}{4}$
4. $1 \frac{5}{8} \div 4$ $\qquad$
5. $8 \frac{1}{6} \div 2 \frac{1}{3}$
6. $5 \div \frac{1}{5}$

## Minute Math Week 2:



1. $\frac{12}{2} \cdot \frac{1}{3}=$
2. Use the correct symbol $(=,>$, or $<)$ to complete: $\frac{3}{10}+\frac{7}{10} \square \frac{3}{10} \cdot \frac{7}{10}$
3. Which of the following does not belong? Circle your answer.

Two-tenths
0.2

20\%

4. The distance between two cities would most likely be measured in:
a. feet
b. inches
c. yards
d. miles
5. The shaded area in figure B is $\qquad$ times greater than the shaded area in figure A .
A

B

6. The perimeter around the shaded area in figure A in Problem 5 is $\qquad$ units.
7. In the graph, $\qquad$ has five times as much money as $\qquad$ .

For Problems 8-10, evaluate if $a=4, b=6$, and $c=2$.
8. $a b=$

9. $\frac{a+b}{c}=$
10. $b^{2}=$

## Week 3:

## Part 1:

## - Two-Step Word Problems

- Average, Part 1
- Two-step word problems can be written as two-step computation problems.

Example: $10-(6+3)$
Parentheses can make the problem easier.
Example: Julie went to the store with $\$ 20$. She bought 8 cans of dog food for $67 \$$ per can. How much money did she have left?

$$
\$ 20-(0.67 \times 8)
$$

1. Find out how much she spent.

$$
0.67 \times 8=\$ 5.36
$$

2. Then find out how much money she had left.

$$
\$ 20-\$ 5.36=\$ 14.64
$$

- Calculating an average is often a two-step process.

1. Add the items.
2. Divide by the number of items.

The answer must be between the smallest and the largest numbers.
Another name for average is mean.
Example: There were 3 people in the first row, 7 in the second row, and 20 in the third row. What was the average number of people in each of the rows?

$$
\begin{aligned}
3 \text { people } \\
7 \text { people }
\end{aligned} \quad \begin{aligned}
& 10 \text { people per row } \\
& +\quad 20 \text { people }
\end{aligned} \quad 3 \text { rows } \begin{aligned}
30 \text { people }
\end{aligned}
$$

- The average of two numbers is the number halfway between the given numbers.


## Practice:

1. Hilda's scores on five games were the following:

83, 89, 94, 99, and 100 . What was her average score? $\qquad$
2. Myrna bought 7 pounds of meat for a barbecue. She paid $\$ 2.49$ per pound and gave the clerk a $\$ 20$ bill. How much change should she receive? $\qquad$
3. Jim drove 175 miles in 3 hours 30 minutes. How many miles per hour did Jim drive?
4. What is the average (mean) of 250 and 450? $\qquad$

Part 2:

## - Order of Operations

- When more than one operation occurs in the same expression, perform the operations in the order listed below.

| Order of Operations |
| :--- |
| 1. Parentheses, brackets, or braces |
| 2. Exponents (powers) and roots |
| 3. Multiply and divide, in order, left to right. |
| 4. Add and subtract, in order, left to right. |

- Another good way to remember the order of operations is with the sentence "Please excuse my dear Aunt Sally." Each initial letter stands for an order-of-operations word.

Parentheses
Exponents
Multiplication and Division

- A division bar may serve as a symbol of inclusion.

Simplify above and below the bar before dividing.
Example: $\frac{3^{2}+3 \cdot 5}{2}=\frac{9+15}{2}=\frac{24}{2}=12$

## Practice:

Simplify 1-4.

1. $12+52-9 \div 3$
2. $(100-7 \cdot 2+3) \div 10$ $\qquad$
3. $\frac{6 \div 2+3 \cdot 9}{\sqrt{100}}$
4. $\frac{4^{2} \cdot 4^{3}+3 \cdot 4}{4}$
5. Evaluate $\frac{\left(a^{2}+a\right) \cdot b}{\sqrt{c}}$ if $a=5, b=3$, and $c=4$.

## Minute Math Week 3:


3. Which of these represents the greatest amount?

| Circle: | $62 \%$ | $\frac{1}{2}$ |
| :--- | :--- | :--- |


4. Use $\bullet,+,-$, or $\div$ to complete the following equation. $2 \square 4 \square 1=9$
5. How many cubes are in this set? $\qquad$

6. The distance around the world at the equator is about 42,000 $\qquad$ .
a. meters
b. kilometers
c. centimeters
d. millimeters
7.

What number will complete the box? $\qquad$


For Problems 8-10, use $>,<$, or $=$.
\&. $50 \%-\frac{1}{2}$
9. $3^{2}$ $\qquad$ $2^{3}$
10.
$0 . \overline{5}$ $\qquad$ 0.5

## Week 4:

## Part 1:

## - Ratio Word Problems

- To solve ratio word problems:

1. Make and complete a ratio box.

Write given numbers in boxes.
Write a letter in the box that answers the question asked.
2. Write a proportion using the numbers in the ratio box.
3. Solve the proportion.

Cross-multiply.
Divide by known factor.
Example: The ratio of salamanders to frogs was 5 to 7 .
If there were 20 salamanders, how many frogs were there?

| Salamanders Frogs | Ratio | Actual Count |
| :---: | :---: | :---: |
|  | 5 | 20 |
|  | 7 | $F$ |
|  | $\underset{\text { frogs }}{\text { alamand }}$ | $=\frac{5}{7}=\frac{20}{F}$ |
|  |  | $\begin{aligned} 5 \cdot F & =7 \cdot 20 \\ 5 F & =140 \end{aligned}$ |
|  |  | $F=\frac{140}{5}$ |
|  |  | $F=28$ frogs |

## Practice:

1. The ratio of bats to balls was 8 to 20 .

If there were 20 bats, how many balls were there?
2. The bread recipe calls for 8 cups of flour and 3 eggs.

The baker used 24 cups of flour to make bread.
How many eggs did she use?
3. The ratio of violin players to cello players was 6 to 3 .

The orchestra had 8 violin players.
How many musicians played the cello?

Part 2:

## - Rate Word Problems

- Rate is a ratio of two measurements.
- Rate can be stated in two ways-as a ratio and its reciprocal.
- Two ways to solve a rate problem:

1. Multiply by the correct form of the rate.

Choose the rate that allows you to cancel the units you want to change and to keep the units you want in your answer.
2. Use the loop method.

Example: Eight ounces of the solution cost 40 cents.
Find the cost of 32 ounces of the solution.

1. Rate method:

The two rates are $\frac{8 \text { ounces }}{40 \text { cents }}$ and $\frac{40 \text { cents }}{8 \text { ounces }}$.
Use the rate that has cents in the numerator to change to cents.
Cancel and multiply.
32 ounces $\times \frac{40 \text { cents }}{8 \text { ounces }} 160$ cents $=\$ 1.60$
2. Loop method:

$$
\frac{\text { ounces }}{\text { cents }} \frac{8}{40}=\frac{32}{?} \longrightarrow(40 \times 32) \div 8=160 \text { cents }=\$ 1.60
$$

## Practice:

1. Eggs cost $\$ 5.07$ for 3 dozen. What was the price per dozen?
2. If 30 pounds of bird seed cost $\$ 21$, how much would 50 pounds cost at the same rate?
3. In Rosa's collection, the ratio of rings to bracelets is 2 to 3 . The ratio of bracelets to necklaces is 2 to 6 . If Rosa has 10 rings, how many necklaces does she have? (Hint: First find how many bracelets she has.)

## Minute Math Week 4:

1. $0.7 \times 8=$
2. $576 \div 10=$
3. If $\frac{2}{5}+\frac{x}{5}=\frac{7}{5}$, then $x=$ $\qquad$ $-$
4. If $\left[\frac{3}{8}\right] \cdot\left[\frac{a}{2}\right]=\frac{15}{16}$, then $a=$ $\qquad$ -
5. In the graph, shade column A and put an $X$ in E4.

6. What shape would the net to the right create if you folded it?
a.

b.

c.

d.



7
About what percent of the graph does region A represent?
a. $50 \%$
b. $90 \%$
c. $10 \%$
d. $33 \%$

For Problems 8-10, estimate to find the best answer.

8. 19 out of 80 :
a. $10 \%$
b. $40 \%$
c. $25 \%$
d. $75 \%$
9.
$9 \%$ of 55 :
a. 50
b. 30
c. 20
d. 5
10. $194 \%$ of 40 :
a. 225
b. 75
c. 40
d. 30

## Week 5:

Part 1:

## - Perimeter

- Perimeter is the distance around a polygon.
- To find perimeter, add the lengths of all the sides.

Example: Find the perimeter of the polygon below.
If some sides are not labeled, add or subtract to find labels for these sides.
(Hint: It helps to use two different colors.)
Trace over all horizontal lines in one color.
Trace over all vertical lines in another color.


## Practice:

1. If one side of a square is 12 inches long, what is the perimeter? $\qquad$
2. The perimeter of a regular triangle equals the perimeter of a square.

The side of the square is 24 inches long. How long is each side of the triangle?
3. If each side of a regular hexagon is $4 \frac{1}{2} \mathrm{in}$., what is its perimeter? $\qquad$
Find the perimeter of each figure below.
4.

5.

6.


## Part 2:

## - Area of a Triangle

- Rectangular Area, Part 2


A triangle has three sides, and any side can be the base. A triangle may have three base-height orientations, as shown by rotating these triangles.

Area of a triangle $=\frac{1}{2} b h$ or $\frac{b h}{2}$

- To find the area of a complex shape:

1. Divide the shape into rectangular parts.
2. Find the area of each part.
3. Add the areas to find the total area.

Sometimes subtracting a "ghost" area (the area that is missing) from a larger rectangle that includes the entire figure is easier.

## Example:



There are three ways to find the area of this shape.


## Practice:

Find the area of each figure.
1.

2.

3.


## Minute Math Week 5:


2. $3+2 \cdot 4+5=$
3. Which of these represents the least amount?

Circle: 0.35

$$
\frac{12}{50}
$$

$$
25 \%
$$


4. Fill in the remaining prime numbers that are less than 20.

5. Shade row 3 and column C.

6. At what point does the row and column shaded in Problem 5 intersect? $\qquad$
7. In 1933, Wiley Post flew around the world in 7 days, 18 hours. Wiley's trip would best be described as flying around the $\qquad$ of the earth.
a. perimeter
b. area
c. volume
d. diameter
8.

Find the number that completes the following problem.

$$
\begin{aligned}
& 2 \square \\
& \times 8 \\
& \hline 192
\end{aligned}
$$

9. Find the number that completes the following problem.

$$
(3+5)+2=2(\square+2)
$$

10. 

If $3 \times 3 \times 3 \times 3=3^{x}$, then $x=$ $\qquad$ .

## Week 6:

## Part 1:

## - Proportions

- A proportion is a statement that two ratios are equal.

Example: $5 \cdot 16=80 \underset{-\frac{16}{20}==<\frac{4}{5} \rightarrow}{ } 20 \cdot 4=80$
Solve a proportion by finding the missing term.

1. Find the cross products.
2. Divide the known product by the known factor.

Example: $\quad \frac{3}{5}=\frac{6}{W}$

$$
\begin{aligned}
3 \cdot W & =5 \cdot 6 \\
3 W & =30 \\
W & =\frac{30}{3} \\
W & =10
\end{aligned}
$$

## Practice:

Solve 1-6.

1. $\frac{16}{4}=\frac{0.4}{r}$
2. $\frac{7}{20}=\frac{a}{40}$ $\qquad$
3. $\frac{24}{32}=\frac{21}{t}$ $\qquad$
4. $\frac{1.5}{2.5}=\frac{b}{45}$ $\qquad$
5. $\frac{0.45}{8.1}=\frac{c}{2.7}$
6. $\frac{w}{50}=\frac{28}{70}$

## Part 2:

- Powers of 10
- The exponent of a power of 10 tells the number of zeros in standard form.

Example: $10^{4}=10,000$

- To multiply powers of 10 , add the exponents.

Example: $10^{3} \times 10^{4}=10^{3}+{ }^{4}=10^{7}$

- To divide powers of 10, subtract the exponents.

Rules of Exponents (for all powers with the same base)

$$
\begin{aligned}
a^{x} \cdot a^{y} & =a^{x}+y \\
\frac{a^{x}}{a^{y}} & =a^{x}-y \\
\left(a^{x}\right)^{y} & =a^{x y}
\end{aligned}
$$

Example: $10^{6} \div 10^{2}=10^{6}-{ }^{2}=10^{4}$

| Trillions |  |  | Billions |  |  | Millions |  |  | Thousands |  |  | Units (Ones) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|l} \hline \text { os } \\ \text { de } \\ \text { D } \\ \text { c } \end{array}$ | $\stackrel{\curvearrowleft}{\Phi}$ | $\begin{aligned} & \text { © } \\ & \text { © } \end{aligned}$ |  | $\stackrel{\oplus}{\oplus}$ | $\begin{aligned} & \mathscr{0} \\ & \stackrel{0}{\circ} \end{aligned}$ |  | $\stackrel{\sim}{\oplus}$ | $\begin{aligned} & \text { ® } \\ & \stackrel{0}{\circ} \end{aligned}$ | $\begin{aligned} & \text { og } \\ & \text { D } \\ & \text { D } \\ & \text { C } \end{aligned}$ | $\stackrel{\oplus}{\Phi}$ | $\begin{aligned} & \stackrel{0}{0} \\ & \check{0} \end{aligned}$ | O 0 0 0 0 5 | $\stackrel{\sim}{\Phi}$ | $\begin{aligned} & \mathscr{\infty} \\ & \stackrel{\circ}{\circ} \end{aligned}$ |
| $10^{14}$ | $10^{13}$ | $10^{12}$ | $10^{11}$ | $10^{10}$ | $10^{9}$ | $10^{8}$ | $10^{7}$ | $10^{6}$ | $10^{5}$ | $10^{4}$ | $10^{3}$ | $10^{2}$ | $10^{1}$ | $10^{\circ}$ |

- Powers of 10 are used in expanded notation.

Example: Write 5206 in expanded notation using powers of 10.

$$
\begin{aligned}
& (5 \times 1000)+(2 \times 100)+(6 \times 1) \\
& \left(5 \times 10^{3}\right)+\left(2 \times 10^{2}\right)+\left(6 \times 10^{\circ}\right)\left(\text { Remember that } 10^{\circ}=1 .\right)
\end{aligned}
$$

- To multiply by a positive power of 10 :

Shift the decimal point to the right the number of places shown by the exponent.

Example: $3.14 \times 10^{4}=\underbrace{31,400}_{4 \text { places }}$

- To divide by a positive power of 10 :

Shift the decimal point to the left the number of places shown by the exponent.

Example: $3.5 \div 10^{4}=\underbrace{0.00035}_{4 \text { places }}$

## Practice:

Solve 1 and 2.

1. $0.238 \times 10^{3}$ $\qquad$ 2. $1.536 \div 10^{2}$ $\qquad$

Write each missing exponent.
3. $\left(10^{6}\right)\left(10^{2}\right)=10^{\square}$ $\qquad$ 4. $\left(7^{7}\right) \div\left(7^{\square}\right)=7^{2}$ $\qquad$
5. Write 1294 in expanded notation using powers of 10.

## Minute Math Week 6:



For Problems 4-6, use the Venn diagram to the right.

$\qquad$ people liked vanilla only.
5. $\qquad$ people liked chocolate only.
6. $\qquad$ people liked both.

For Problems 7-10, circle True or False.
2. $\frac{8}{8}>\frac{12}{12}$

True or False

巴. $\frac{12}{50}=\frac{6}{25}$
True or False

อ. $2.2>2.0 \overline{9}$
True or False
10.

$$
8.15=8+\frac{1}{10}+\frac{5}{100}
$$

True or False

## Enjoy the rest of your summer!

